Ref. No.: Ex/MATH/CSE/T/211/2016

B.CSE, 2ND YR. 1ST SEMS EXAM, 2016

Mathematics

(Paper-IV)

Full Marks:100

Time: Three Hours

**Urs**

Answer Question number 1. and any six from the rest.

1. Find a particular integral of the differential equation

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day - 9y = exacos a

2. (a) Find the series for log(1+x) by integration and use Abel's Theorem to prove that

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*......... = log2*

4T

(b) Find a power series solution of the initial value problem

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dyLTY

(22 – non solo pet oy = 0, 2(0) = 4, (0) = 6

(2

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1)

*dir*

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+ xy = 0,

y(0) =

4,

y'(0) = 6

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Write atleast first five terms of the series. 3. (a) Find Frobenius series solution about the regular singular point of the following

differential equation

edy + 8(22 – 1)y = 0

dx2 dx Write atleast first three terms of each series. (b) State the orthogonality property of Chebyshev ploynomials of first kind. Use that

property to find the expansion of f(x) = x3 + x,-1 < x < 1 in terms of the

Chebyshev polynomials of first kind. 4. (a) Prove that

*,min Pm (2) Pn(x) dx =*

J-1 mli)fnler 1 2niti ,m=n where Pn(x) is the Legendra polynomial of degree n. (b) Write generating function of Legendre ploynomials. Use that function to prove

i. Pn(1) = 1 ii. Pan (0) = (–1)n 1.3.5mmmm(2n-1)

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2n+1

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.3.5...

2nn!

5. (a) Use the method of variation of parameters to find general solution of the equation

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*+y = tan x*

(b) Solve

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dy + 49

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*9 + 4y = 2x In x*

*dira tu*

*da*

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(4)

6. (a) If f(x) = e, describe the image under f(x) of horizontal and vertical lines i.e. find

the sets f(a+it) and f(t + ib), where a,b are constants and t runs through all real

numbers. (b) If the function analytic in its domain of definition? (c) Suppose f(x) = aza + bzz + cza, where a, b, c are fixed complex numbers. By

differentiating f(2), show that f(2) is complex differentiable at z iff bz + 2cz = 0. (d) Derive the polar form of the Cauchy-Riemann equations for u and v: one =

mot 7. (a) Use Liouville's theorem to prove that every polynomial in z of degree n(> 1) has a

zero. (b) Find harmonic conjugate of xy + 3x2y – 43. (c) Define u(x) = Im(+) for z #0 and set u(0) = 0 , then show that

i. venner og = 0. ii. u is not harmonic on C. iii. com does not exists at (0,0).

3 C

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***e***

***v***

8. (a) Find

f(z)dz where v = 3eit for t € (0,27] and f(x) = 2. (b) Show that if zo is an isolated singularity of f(z) that is not removable, then zo is

an essential singularity of ef(z), (c) By estimating the coefficient of the Laurent series, prove that if zo is an isolated

singularity of f , and if (z - 20) f() +0 as 2 + zo, then zo is removable.

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9. (a) Define Fourier series of a function f(x). Find the Fourier series generated by a

periodic function f(x) = x2 in -< x <a and deduce that

i. 1 + 1 + 32 + ... =

ii. 11 +31 + 2 + ... = (b) Find the Fourier series for f(x) = |x|, - a< <a

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